Sanjay Ghodawat University, Kolhapur

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2019-20
EXM/P/09/00

Year and Program: F.Y.B.Sc | **School of Science** Section-A. | **Department : Mathematics**

Course Code – MTS 101 | **Course Title – Mathematics-I** | **Semester – I**

Day and Date – Friday 6/12/19 | **End Semester Examination** | **Time:** 1½ hr 10.30am to 11.am

Max Marks: 100

PRN number – | **Seat no-** | **Answer Booklet No.-**

Students' Signature - | **Invigilator's Signature –**

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question is followed by four plausible alternatives, Tick (√) the correct one.
4) Answer to question 1 should be written in the question paper and submit to the Jr. Supervisor.
5) If you tick more than one option it will not be evaluated
6) Figures to the right indicate full marks
7) Use **Blue ball pen** only.

Q.1	**Tick mark (√) correct alternative.**	Marks	Bloom's Level	CO
i)	A function f is such that $\lim_{x\to c} f(x)$ exists but not equal to $f(c)$, then the discontinuity is called a) removable, c) non-removable, b) mixed, d) oscillatory.	1	L1	CO1
ii)	If $f(x)=\lvert x\rvert$, then $f(x)$ is _____ at $x=0$. a) continuous, c) discontinuous, b) derivable, d) oscillate.	1	L2	CO1
iii)	Which one of the following is a mean value theorem? a) Leibnitz's Theorem, c) Euler's Theorem, b) Taylor's Theorem, d) Rolle's Theorem.	1	L1	CO2
iv)	If $f(x)=e^x$, $x\in[0,1]$ then the value of c by Lagrange's mean value theorem is a) $e-1$, c) $\frac{1}{e-1}$, b) $e(e-1)$, d) $\log(e-1)$.	1	L3	CO2

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v) If $y = e^{3x}$, then n^{th} derivative of y is — 1 L3 CO3

a) $3^n e^x$, c) $3^n e^3$,

b) $3e^x$, d) $3^n e^{3x}$.

vi) If $y = x^n$, then $(n+1)^{th}$ derivative of y is — 1 L2 CO3

a) $(n+1)!$, c) $n!$,

b) 0, d) $(n-1)!$.

vii) The infinite series $1 - x + x^2 - x^3 + \cdots$ is the expansion of — 1 L1 CO4

a) $\frac{1}{1+x}$, c) $\frac{1}{1-x}$,

b) e^x, d) e^{-x}.

viii) The infinite series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ is the expansion of — 1 L1 CO4

a) $\sin x$, c) $\cos x$,

b) $\sinh x$, d) $\cosh x$.

ix) $\lim_{x \to 1} \frac{\sin \pi x}{x-1}$ is equal to — 1 L3 CO5

a) 1, c) -1,

b) π, d) $-\pi$.

x) $\lim_{x \to a} \frac{\cos x - \cos a}{x-a}$ is equal to — 1 L3 CO5

a) $-\sin a$, c) $\cos a$,

b) $\sin a$, d) $-\cos a$.

xi) $\lim_{x \to 3} \frac{\log x - \log 3}{x-3}$ is equal to — 1 L3 CO5

a) $\frac{2}{3}$, c) 3,

b) $\frac{1}{3}$, d) $\frac{-1}{3}$.

xii) $\lim_{x \to a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$ is equal to — 1 L3 CO5

a) $\sqrt{2} \sin a$, c) $2\sqrt{a} \cos a$,

b) $\sqrt{2a} \sin a$, d) $\sqrt{2a} \cos a$.

xiii) $\lim_{x\to 0}\frac{1-\cos mx}{x^2}$ is equal to 1 L3 CO5

a) $\frac{m}{2}$, b) $\frac{m^2}{2}$, c) $\frac{2}{m}$, d) $\frac{2}{m^2}$.

xiv) $\lim_{x\to 0}\frac{e^{2x}-1}{e^{3x}-1}$ is equal to 1 L3 CO5

a) $\frac{3}{2}$, b) $\frac{2}{3}$, c) $\frac{4}{9}$, d) $\frac{9}{4}$.

xv) One of the values of $\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)^3$ is equal to 1 L2 CO6

a) 2, b) 1, c) -1, d) 0,

xvi) The value of $\frac{(\cos 3x+i\sin 3x)^5}{(\cos 2x-i\sin 2x)^{-4}}$ is equal to 1 L2 CO6

a) $\cos 3x+i\sin 3x$, c) $\cos 7x+i\sin 7x$,

b) $\cos 9x+i\sin 9x$, d) $\cos x+i\sin x$.

xvii) One of the values of $\left(\sin\frac{\pi}{3}+i\cos\frac{\pi}{3}\right)^6$ is equal to 1 L2 CO6

a) 2, b) 1, c) -1, d) -2.

xviii) If two complex numbers z_1 and z_2 are such that $z_1+z_2=0$ then $\frac{1}{z_1}+\frac{1}{z_2}$ is equal to 1 L1 CO6

a) 1, b) 0, c) -1, d) i.

xix) For any complex number z, $\sinh^{-1}z$ is equal to 1 L1 CO6

a) $\log\left(z-\sqrt{z^2-1}\right)$, c) $\log\left(z+\sqrt{z^2-1}\right)$,

b) $\log\left(z-\sqrt{z^2+1}\right)$, d) $\log\left(z+\sqrt{z^2+1}\right)$.

xx) The value of $\tan^{-1}(\cos\pi)$ is equal to 1 L2 CO6

a) $\frac{\pi}{2}$, b) $\frac{-\pi}{2}$, c) $\frac{-\pi}{4}$, d) $\frac{\pi}{4}$.

Year and Program: **School of Science** **Department of Mathematics**
F.Y.B.Sc
Course Code: MTS 101 **Course Title: Mathematics-I** **Semester – I**
Day and Date: Friday 6/12/19 **End Semester Examination (ESE)** **Time:** 2.5hr 11am to 1.30pm **Max Marks: 100**

Instructions:
1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Non-programmable calculator is allowed

			Marks	Bloom's Level	CO
Q.2	A)	**Solve any Two of the following.**			
	i)	If $\lim_{x\to a} f(x) = l_1$ and $\lim_{x\to a} g(x) = l_2$ then show that $\lim_{x\to a}\big(f(x)+g(x)\big) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$.	4	L1	CO1
	ii)	Let $f(x) = x^2$. Prove that $x^2 \to 0$ as $x \to 0$.	4	L2	CO1
	iii)	Use $\varepsilon - \delta$ definition of limit to prove that $\lim_{x\to 2}(3x-5) = 1$.	4	L2	CO1
	B)	**Solve any One of the following.**			
	i)	State and prove Lagrange's mean value theorem.	8	L1	CO2
	ii)	State and prove Rolle's theorem.	8	L1	CO2
Q.3	A)	**Solve any Two of the following.**			
	i)	If $y = \frac{1}{ax+b}$, then show that $y_n = \frac{(-1)^n n! a^n}{(ax+b)^{n+1}}$.	4	L2	CO3
	ii)	If $y = a^{mx}$, then show that $y_n = m^n a^{mx} (\log a)^n$.	4	L2	CO3
	iii)	Find the n^{th} derivative of $x^3 cosx$.	4	L2	CO3
	B)	**Solve any one of the following**			
	i)	State and prove Maclaurin's theorem.	8	L1	CO4
	ii)	Using Taylors theorem express $(x-2)^4 - 3(x-2)^3 + 4(x-2)^2 + 5$ in powers of x.	8	L2	CO4

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Q.4		**Solve any four of the following**			
	i)	Evaluate $\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2}$.	4	L3	CO5
	ii)	Evaluate $\lim_{x\to 0} \frac{\log \tan x}{\log x}$.	4	L3	CO5
	iii)	Evaluate $\lim_{x\to 0} \sin x \log x$.	4	L3	CO5
	iv)	Evaluate $\lim_{x\to 1}\left[\frac{1}{\log x} - \frac{x}{x-1}\right]$.	4	L3	CO5
	v)	Evaluate $\lim_{x\to 0}\left(a^x + x\right)^{\frac{1}{x}}$.	4	L3	CO5
	vi)	Evaluate $\lim_{x\to 0}\left(\frac{1}{x}\right)^x$.	4	L3	CO5
Q.5	A)	**Solve any one of the following**			
	i)	Simplify $\frac{(\cos 4\theta - i\sin 4\theta)^2 (\cos 5\theta - i\sin 5\theta)^3}{(\cos 3\theta + i\sin 3\theta)^7 (\cos 8\theta - i\sin 8\theta)^5}$.	6	L2	CO6
	ii)	Express $\tan 3\theta$ in terms of $\tan\theta$.	6	L2	CO6
	B)	**Solve any Two of the following**			
	i)	Find all the 3^{rd} roots of unity.	5	L3	CO6
	ii)	Solve the equation $x^4 + 1 = 0$.	5	L3	CO6
	iii)	If $\sin(\alpha + i\beta) = x + iy$, then prove that $\frac{x^2}{\cosh^2\beta} + \frac{y^2}{\sinh^2\beta} = 1$.	5	L3	CO6
Q.6	A)	**Solve any One of the following**			
	i)	Find a, b, c if $\lim_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$.	8	L4	CO5
	ii)	If $\lim_{x\to 0} \frac{\sin 2x + a\sin x}{x^3}$ is finite, find the value of a and hence the limit.	8	L4	CO5

B) **Solve any One of the following**

i)	Let n be a integer and θ be any real number, then show that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.	8	L1	CO6
ii)	For any real value of x, show that	8	L2	CO6
	a) $\cosh^{-1} x = \log\left(x + \sqrt{x^2 - 1}\right)$, $x \geq 1$.			
	b) $\tanh^{-1} x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$ for $\lvert x\rvert < 1$.			

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